Thermal conductivity anisotropy of metasedimentary and igneous rocks

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Thermal conductivity anisotropy was determined for three sets of metasedimentary and igneous rocks from central Utah, USA. Most conductivity measurements were made in transient mode with a half-space, line source instrument oriented in two orthogonal directions on a flat face cut perpendicular to bedding. One orientation of the probe yields thermal conductivity parallel to bedding \( (k_{\text{par}}) \) directly, the other orientation of the probe measures a product of conductivities parallel and perpendicular to bedding from which the perpendicular conductivity \( (k_{\text{perp}}) \) is calculated. Some direct measurements of \( k_{\text{par}} \) and \( k_{\text{perp}} \) were made on oriented cylindrical discs using a conventional divided bar device in steady state mode. Anisotropy is defined as \( k_{\text{par}}/k_{\text{perp}} \). Precambrian argillites from Big Cottonwood Canyon have anisotropy values from 0.8 to 2.1 with corresponding conductivity perpendicular to bedding of 2.0 to 6.2 W m\(^{-1}\) K\(^{-1}\). Anisotropy values for Price Canyon sedimentary samples are less than 1.2 with a mean of 1.04 although thermal conductivity perpendicular to bedding for the samples varied from 1.3 to 5.0 W m\(^{-1}\) K\(^{-1}\). The granitic rocks were found to be essentially isotropic with thermal conductivity perpendicular to bedding having a range of 2.2 to 3.2 W m\(^{-1}\) K\(^{-1}\) and a mean of 2.68 W m\(^{-1}\) K\(^{-1}\). The results confirm the observation by Deming [1994] that anisotropy is negligible for rocks having \( k_{\text{perp}} \) greater than 4.0 W m\(^{-1}\) K\(^{-1}\) and generally increases for low conductivity metamorphic and clay-rich rocks. There is little evidence, however, for his suggestion that thermal conductivity anisotropy of all rocks increases systematically to about 2.5 for low thermal conductivity rocks.


1. Introduction

Many laminated metamorphic and sedimentary rocks conduct heat preferentially in a direction parallel to the bedding planes of these rocks. This thermal conductivity anisotropy can be important in determining upper crustal thermal regimes, especially those of sedimentary basins where laminated rock are volumetrically significant. Thermal conductivity anisotropy reported in the literature varies from 0.99 to 4.7 [Kappelmeyer and Haenel, 1974; Grubbe et al., 1983], thermal conductivities perpendicular to bedding for the same set of samples varies between 0.29 and 7 W m\(^{-1}\) K\(^{-1}\) [Zoth and Haenel, 1988].

Thermal conductivity anisotropy is a function of rock mineralogy and fabric [Pribnow and Umsonst, 1993], particularly the bedding planes of a rock. Thermal conductivity anisotropy is especially pronounced in shales and clay-rich rocks. This anisotropy is reported to be a result of the presence of anisotropic mica minerals whose conductivity parallel to the c axis is much less than the conductivity perpendicular to it. These mica minerals tend to lie in the bedding planes of rocks with the c axis oriented perpendicular to bedding; thus, the anisotropy of the minerals is transferred collectively to the anisotropy of the whole rock.

Deming [1994] has suggested an empirical relationship that relates thermal conductivity anisotropy and thermal conductivity perpendicular to bedding. In Deming’s model, thermal conductivity is isotropic for \( k_{\text{perp}} > 4 \) W m\(^{-1}\) K\(^{-1}\), but anisotropy increases with decreasing \( k_{\text{perp}} \), rising to a value of 2.5 when \( k_{\text{perp}} \) is 1 W m\(^{-1}\) K\(^{-1}\). Deming’s relationship is based on measurements on 89 rock samples gleaned from the literature [Deming et al., 1992; Grubbe et al., 1983; Kappelmeyer and Haenel, 1974]. If a unique relationship exists, it could be used in sedimentary basins to make corrections for an effective vertical thermal conductivity, especially where only borehole cuttings are available for thermal conductivity measurements.

The purpose of this paper is to, first, report new values of thermal conductivity anisotropy and thermal conductivity perpendicular to bedding. Twenty-one samples of laminated sedimentary rocks were obtained from central Utah’s Price City, Utah, USA.
2. Thermal Conductivity Measurements

Thermal conductivity measurements were made with two different instruments. Most measurements were made in transient mode with a half-space line source device. Thermal conductivity, determined by the line source method, is a scalar quantity representing conductivity in the plane perpendicular to the line source axis. The German-built TeKa TK04 thermal conductivity meter employs a cylindrical needle probe 71 mm in length and 3 mm in diameter, partially encased in a clear plastic cylinder 88 mm in diameter (Figure 1). The line source is heated with constant power while the source temperature sensed at the midpoint of the needle is simultaneously recorded. Each measurement made with the TK04 consists of an 80-s heating period followed by calcula-

![Image](image_url)
tion of the conductivity over a specified time interval and then a measurement of the temperature drift of the system before the next heating period resumes. The TK04 can make up to 99 successive repeat measurements for a given sample setup.

[7] Repeated measurements were made to improve precision. We have observed a typical equilibration period at the onset of measurements on a sample in which individual measurements have scatter up to 20% from the mean, followed by a long, stable period when individual measurements are within 5% of the mean. Consequently, a total of 10 to 15 measurements were made on each sample and the conductivity values reported in this paper are a result of the average of at least six measurements generally from between measurements five and ten. The average thermal conductivity values reported for each sample have typical standard deviations from 0.01 to 0.30 W m$^{-1}$ K$^{-1}$, with most less than 0.10 W m$^{-1}$ K$^{-1}$. Some of the quartzite samples have larger standard deviations (up to ~15%) most likely due to the orientation of the quartz grains in the sample [Pribnow and Umsonst, 1993]. Standard errors of the mean are less than 2% for the majority of conductivity values reported in this paper.

[8] All of the rock samples were cut perpendicular to the bedding plane, and lapped to a flat surface. The flat surfaces were scribed with two perpendicular lines or full grids to facilitate the position and orientation of the probe on the sample face. All samples were saturated prior to measurement by placing in a vacuum for a minimum of two hours and then saturating with tap water for approximately 18 to 24 hours before any probe measurements were made.

[9] The second type of thermal conductivity measurements used a divided bar device [Roy et al., 1968; Sass et al., 1971; Chapman et al., 1981] in steady state mode. Cores, 47 millimeters in diameter, were drilled with axis parallel and perpendicular to bedding respectively and cut into discs about 10 millimeters thick. Samples were saturated with tap water prior to measurement. Measurement precision for the divided bar is about 2%; accuracy gauged by interlaboratory comparison and measurement on standards is generally better than 5% [Chapman, 1976].

3. Thermal Conductivity Anisotropy

[10] The line source probe measures conductivity in a plane perpendicular to the axis of the needle probe. As all samples have been cut perpendicular to bedding, the placement of the probe axis perpendicular to bedding on the sample surface measures the conductivity parallel to bedding ($k_{par}$). Placement of the probe axis parallel to bedding measures a minimum conductivity ($k_{min}$) that can be measured with the line source, including components of conductivity both perpendicular and parallel to bedding. Thermal conductivity perpendicular to bedding is found from the relationship [Pribnow and Sass, 1995]

$$k_{perp} = (k_{min})^2 / k_{par}.$$  (1)

With the line source probe axis at intermediate angles to bedding, $\gamma$, an apparent thermal conductivity $k_{app}$ is given by Pribnow (personal communication)

$$k_{app} = (k_{par} \sin^2 \gamma + k_{par} k_{perp} \cos^2 \gamma)^{1/2}.$$  (2)

This relationship yields $k_{app} = k_{par}$ for $\gamma = 90^\circ$ and $k_{app} = k_{min} = (k_{par} k_{perp})^{1/2}$ for $\gamma = 0^\circ$. The thermal conductivity anisotropy, $A$, of a given rock is defined as

$$A = k_{par}/k_{perp}.$$  (3)

4. Observations and Results

[11] We first sought to test the general model of the measurement of thermal conductivity anisotropy with the line source device. The test consists of measuring the variation of apparent thermal conductivity as a function of the orientation of the needle-probe axis with the bedding plane. These orientations ranged from 0° to 90°, increased in 10° to 15° increments. Results for samples BC-9701, BC-9702, and BC-9806a, laminated argillites from the Precambrian Big Cottonwood Formation, are shown in Figure 2. Sample BC-9806a (Table 1) has a systematic change of apparent conductivity with needle probe orientation, from a minimum value less than 3.9 W m$^{-1}$ K$^{-1}$ with the probe parallel to bedding ($\gamma = 0^\circ$) to a maximum of 4.5 W m$^{-1}$ K$^{-1}$ with the probe perpendicular to bedding ($\gamma = 90^\circ$). The data are reasonably well fit by equation (2); a best fit curve to the data yields 4.56 W m$^{-1}$ K$^{-1}$ for $k_{par}$, 3.95 W m$^{-1}$ K$^{-1}$ for $k_{min}$ and anisotropy $A$ of 1.33. Calculation of apparent thermal conductivity from measurements for this sample at $\gamma = 0^\circ$ and $\gamma = 90^\circ$ [equation (1)] yields an identical anisotropy of 1.33 (Table 1). Likewise thermal conductivity measurements on samples BC-9701 and BC-9702 are reasonably well explained by the geometrical factors inherent in equation (2) (Figure 2).

[12] The thermal conductivity anisotropy results for the collected samples are given in Table 1. The samples are identified by locality and are given with a brief geologic description. Measured values of $k_{par}$ and $k_{min}$ are given; the value of $k_{perp}$ has been calculated from $k_{par}$ and $k_{min}$ and the anisotropy of the sample has been calculated from equation (3) using $k_{par}$ and $k_{perp}$.

[13] Measured $k_{par}$ values range from 1.54 to 7.68 W m$^{-1}$ K$^{-1}$, while $k_{min}$ values range from 1.42 to 8.19 W m$^{-1}$ K$^{-1}$, revealing a similar and broad range of apparent thermal conductivities in mutually perpendicular directions. Calculated values of $k_{perp}$ range from 1.31 to 8.73 W m$^{-1}$ K$^{-1}$, a range shifted slightly lower in magnitude than that of $k_{min}$, as $k_{min}$ includes both $k_{par}$ and some component of $k_{perp}$. Rock samples with high measured values (> 6 W m$^{-1}$ K$^{-1}$) of thermal conductivity were generally quartzites. Exclusion of the quartz-rich rocks lowers calculated values of $k_{perp}$ to a range of 1.31 to 6.18 W m$^{-1}$ K$^{-1}$, with only two samples higher than 5.24 W m$^{-1}$ K$^{-1}$.

[14] The Price Canyon samples are virtually isotropic, with anisotropy ranging from 0.64 to 1.18, with a mean of 1.04. Sample PC-9702, with an anisotropy of 0.64, varies in both composition and bedding characteristics, which may provide an explanation for its unusually low value. The granitic rocks from Little Cottonwood Canyon are also isotropic, with a mean anisotropy of 1.01. The thermal conductivity perpendicular to emplacement orientation (bedding) of these igneous rocks does vary, having a range of 2.2 to 3.2 W m$^{-1}$ K$^{-1}$ and a mean of 2.68 W m$^{-1}$ K$^{-1}$. The mean values of these igneous rocks are very similar to
the results of other granitic rocks from the literature [e.g., Sass et al., 1992; \( k_{\text{perp}} = 2.5 \); anisotropy = 1.1 for samples from Cajon Pass, California]. The quartzites from the Big Cottonwood Formation all had anisotropy values between 0.86 to 0.96 with very high thermal conductivity perpendicular to bedding ranging from 5.88 to 8.73 W m\(^{-1}\) K\(^{-1}\). The Precambrian argillites had a mean anisotropy of 1.25, with only one sample (BC-9701) showing extreme anisotropy with a value of 2.12.

The thermal conductivity of four samples were measured using a divided bar device to test the results of the line source measurements (Table 2). Results from the two methods are not expected to be identical as the methods sample different volumes of rock [Beck, 1988]. Given that caveat, the calculated thermal conductivity anisotropy for these four samples using the two different measurement methods indicate that there is no bias favoring lower (or higher) anisotropy with one method over the other. The same result was concluded comparing the divided bar and line source measurements by Sass et al. [1984]. We assert that use of the line source techniques can produce reliable anisotropy results in an efficient and convenient manner.

Results for thermal conductivity anisotropy \((A)\) are plotted in Figure 3b. The solid line in both Figures 3a and

![Figure 2. Apparent thermal conductivity as a function of probe orientation for samples BC-9701, BC-9702 and BC-9806a. BC-9806a has been offset by a value of 1 Wm\(^{-1}\)K\(^{-1}\) to differentiate it from the other samples. Bars indicate the standard deviation of six or more measurements during one sample determination.](image-url)

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Orientation</th>
<th>(k_{\text{LS}}) (Wm(^{-1})K(^{-1}))</th>
<th>(k_{\text{DB}}) (Wm(^{-1})K(^{-1}))</th>
<th>% Difference</th>
<th>Anisotropy (LS)</th>
<th>Anisotropy (DB)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC-9708</td>
<td>par</td>
<td>3.99</td>
<td>4.18</td>
<td>4.7</td>
<td>0.99</td>
<td>0.92</td>
<td>-7.3</td>
</tr>
<tr>
<td>PC-9708</td>
<td>perp</td>
<td>4.05</td>
<td>4.52</td>
<td>11.0</td>
<td>0.99</td>
<td>0.92</td>
<td>-7.3</td>
</tr>
<tr>
<td>PC-9712</td>
<td>par</td>
<td>4.44</td>
<td>4.83</td>
<td>8.4</td>
<td>1.07</td>
<td>1.40</td>
<td>26.7</td>
</tr>
<tr>
<td>PC-9712</td>
<td>perp</td>
<td>4.15</td>
<td>3.44</td>
<td>-18.7</td>
<td>1.07</td>
<td>1.40</td>
<td>26.7</td>
</tr>
<tr>
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<td>4.20</td>
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<td>1.07</td>
<td>1.40</td>
<td>26.7</td>
</tr>
<tr>
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<td>1.86</td>
<td>-13.1</td>
</tr>
<tr>
<td>BC-9702</td>
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<td>4.84</td>
<td>5.5</td>
<td>2.12</td>
<td>1.86</td>
<td>-13.1</td>
</tr>
<tr>
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<td>3.44</td>
<td>3.25</td>
<td>-5.7</td>
<td>1.33</td>
<td>1.49</td>
<td>11.3</td>
</tr>
</tbody>
</table>
3b is the regression formulated by Deming [1994], which is given by the relationship:

$$A = \alpha \left( \frac{\log k_{\text{perp}} - \log \phi}{\log \lambda_{\text{perp}} - \log \phi} \right),$$

where $A$ is the bulk rock thermal conductivity anisotropy, $\alpha$ is the anisotropy of the end-member component with low thermal conductivity, $k_{\text{perp}}$ is the thermal conductivity of the bulk rock perpendicular to bedding, $\lambda_{\text{perp}}$ is the thermal conductivity perpendicular to the bedding of the low thermal conductivity end-member component, and $\phi$ is the thermal conductivity of the isotropic, high thermal conductivity end-member component. For his regression, Deming [1994] has assigned $\lambda_{\text{perp}} = 1 \text{ W m}^{-1}\text{K}^{-1}$, $\phi = 4 \text{ W m}^{-1}\text{K}^{-1}$, and $\alpha = 2.56$. These same values have been used to plot the regression on Figure 3.
5. Discussion

One unexpected result of these thermal conductivity measurements is the lack of pronounced anisotropy, even though obvious layered fabric is present in all of the sedimentary and metamorphic samples. Only one rock, BC-9701, a shale from the Precambrian Big Cottonwood Formation, reveals distinct anisotropy greater than 1.5. From a preliminary inspection of rock types, the biggest compositional difference of this shale/argillite from the other samples is its lack of any carbonate or carbonaceous content; it is a purely detrital shale. Furthermore, it is much older than the other samples and may have been slightly metamorphosed, although deformation and metamorphic fabric are not visibly present. This observation would tend to indicate that the presence of carbonate might make the samples more isotropic, even when layering is obvious. Limestones and other carbonaceous rocks analyzed in our study are nearly isotropic, with a mean anisotropy of 1.04.

Granitic igneous rocks [this study; also Sass et al., 1992] display no anisotropy. Rocks that are quartz-rich, particularly quartzites, have very high thermal conductivity perpendicular to bedding planes and anisotropy near or slightly less than 1. Quartz typically has high thermal conductivity, but with anisotropy generally greater than 1.5 [e.g., Clauser and Huenges, 1995]. The anisotropy seen in our quartzite samples could be created if there were preferred recrystallization (conductive pathways) in the vertical direction during metamorphism. Metamorphic and clay-rich rocks both generally have pronounced anisotropy. Metamorphic rocks do appear to follow the Deming trend (Figures 3a and 3b), while the clay-rich rocks have a much more narrow range of thermal conductivity perpendicular to bedding and extend from low to very high anisotropy values (Figure 3a). This variability seems to be due to the orientation of sheet silicate minerals in the clays and metamorphic rocks as these minerals have been shown to have high thermal conductivity anisotropy [Diment and Pratt, 1988; Clauser and Huenges, 1995]. Some metamorphic rocks do exhibit high thermal conductivity perpendicular to bedding and are isotropic; these argillites generally have quartz-rich layers. This would imply that quartz-rich metamorphic rocks should be isotropic.

While it is apparent from Figure 3 that Deming’s [1994] regression does fit some of the data, it is also clear that the majority of the data lie below the curve. Our sedimentary rock samples show virtually no thermal conductivity anisotropy (Figure 3b, Table 1). Rocks in the critical conductivity range with \( k_{\text{perp}} \) ranging from 1 to 3 W m\(^{-1}\) K\(^{-1}\) do not remain close to the Deming [1994] proposed relationship. These points have low conductivities perpendicular to bedding and show little anisotropy. Compositionally, most of these samples contain carbon and carbonate, both of which seem to contribute to reduced thermal conductivity and reduced anisotropy. Since coals have a very low thermal conductivity of about 0.33 W m\(^{-1}\) K\(^{-1}\) [Herrin and Deming, 1996], the carbon content would be the reasonable culprit in reducing the overall thermal conductivity of these rocks. The few sedimentary rocks that do have high anisotropy are from the literature [Deming et al., 1992] and are generally of unclear lithology. Most of these samples may be clay-rich shales, furthering the evidence that non-shale sedimentary rocks are isotropic while clay-rich and metamorphic rocks have variable anisotropy.

Fractures, induced in the rock during either coring or sample preparation, could affect the apparent thermal conductivity anisotropy. These fractures generally propagate preferentially to the bedding planes. The thermal conductivity of the fractured rock is affected by the thermal conductivity of the fluid that fills the fracture space [Clauser and Huenges, 1995]. The effective thermal conductivity, \( k_{\text{eff}} \) of a rock, with fractures parallel to the bedding planes, can be determined for both the parallel and perpendicular directions by

\[
\begin{align*}
k_{\text{eff}} &= (k_1 l_1 + k_2 l_2) / (l_1 + l_2),
\end{align*}
\]

and

\[
\begin{align*}
k_{\text{eff}} &= (l_1 + l_2) / [(l_1 / k_1) + (l_2 / k_2)],
\end{align*}
\]

where \( k_1 \) is the thermal conductivity of the rock, \( k_2 \) is the thermal conductivity of the fluid in the fractures, and \( l_1 \) and \( l_2 \) are the thickness of the rock and fractures, respectively. For an isotropic rock of thermal conductivity of 3 W m\(^{-1}\) K\(^{-1}\) and fracture porosity of 1, 5, and 10%, the effective anisotropy is predicted to be 1.03, 1.15, and 1.29, respectively, if the fluid is water \((k = 0.6 \text{ W m}^{-1}\text{ K}^{-1})\). If the samples are dry (air filled fractures; \( k = 0.025 \text{ W m}^{-1}\text{ K}^{-1})\), the effective anisotropy is predicted to be even greater \((2.17, 6.61, \text{ and } 11.62 \text{ for } 1, 5, \text{ and } 10\% \text{ fracture porosity})\). This may help explain the discrepancy between our fully saturated measurements and those used by Deming [1994]. If the literature measurements contained dry or nonsaturated measurements, the increase in anisotropy may be enough to create the apparent trend seen by Deming [1994].

6. Conclusions

Measurement and analysis of thermal conductivity anisotropy in 74 samples of laminated sedimentary and metamorphic rocks and granites from north central Utah lead to the following conclusions:

Line source thermal conductivity instruments afford a rapid and convenient method for determining thermal conductivity anisotropy.

Anisotropy for laminated rocks can be computed from only two measurements on a rock face cut perpendicular to bedding; a more robust estimate of anisotropy can be obtained by fitting a series of measurements on the same rock face made at multiple angles to the bedding direction.

A series of 21 low conductivity (range 0.64 to 1.18) laminated limey and carbonaceous shales and sandstones from Price Canyon have mean anisotropy of 1.04. These samples do not fit the Deming [1994] proposal for a correction for anisotropic effects whereby anisotropy increases systematically from 1.0 to 4.0 with decreasing conductivity perpendicular to bedding.
Anisotropy for high conductivity (range 0.86 to 0.96) laminated quartzites is close to unity.

Thermal conductivity for granitic rocks from Little Cottonwood Canyon is isotropic. Thirty samples yielded tightly clustered anisotropy values with a mean of 1.01 and a standard deviation of 0.02.

While some rocks of low conductivity yielded elevated anisotropy values up to 1.5, Price Canyon samples of low conductivity do not. Thus, a universal correction for thermal conductivity anisotropy sought by Deming [1994] remains elusive.

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